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§12.6 Quadratic Surfaces

Ex: Understand the Surface w/ equation $x^2+y^2-2x-6y-z+10=0$

Solution: First we will rewrite the equation (via completing the square)

$$x^2+y^2-2x-6y-z+10=0$$
$$(x^2-2x)+(y^2-6y)-z+10=0$$
$$(x^2+2(-1)x+(-1)^2-(-1)^2)+(y^2+2(-3)y+(-3)^2-(-3)^2)-z+10=0$$
$$(x-1)^2-(-1)^2+(y-3)^2-(-3)^2-z+10=0$$
$$(x-1)^2+(y-3)^2-z=0$$

Now we analyze the equation via cross-section

when $z=k$ ← (constant):

$$(x-1)^2+(y-3)^2-k=0$$

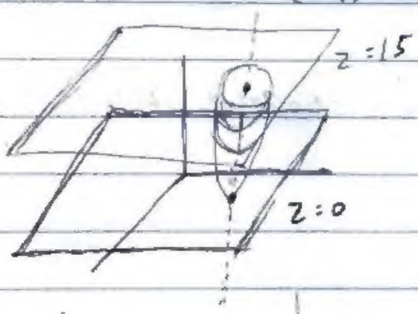
i.e. $(x-1)^2+(y-3)^2=k$ ← (elipse, or point, or empty if k is negative)

when $y=k$: $(x-1)^2+(k-3)^2-z=0$

i.e. $z=(x-1)^2+(k-3)^2$ ← parabola (upward facing)

when $x=k$: $(k-1)^2+(y-3)^2-z=0$

i.e. $z=(k-1)^2+(y-3)^2$ ← parabola (upward facing)



*Conic Sections

Ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Parabolas $\frac{x^2}{a^2}+\frac{y}{c}=0$

Hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

Equation:	Name:
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z}{c}=0$	Elliptic Paraboloid
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Ellipsoid
$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z}{c}=0$	Hypebolic Paraboloid
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	One-sheet Hyperboloid
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	Cone
$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Two-sheet Hyperbola

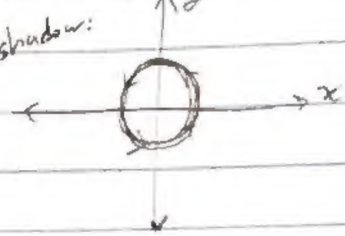
§13.1: Space Curves

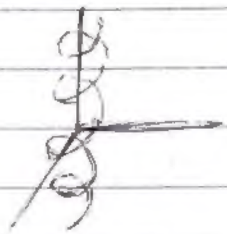
A <u>space curve</u> is a function $\vec{r}: I \to \mathbb{R}^n$

Ex: The helix is the curve

$$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$$

shadow:





Defn: The limit of space curve $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ at time $t = a$ is the component limit provided each component limits as $t \to a$

i.e. $$\lim_{t\to a} \vec{r}(t) = \lim_{t \to a} \langle x(t), y(t), z(t) \rangle$$

$$= \left\langle \lim_{t\to a} x(t), \lim_{t\to a} y(t), \lim_{t\to a} z(t) \right\rangle$$

<u>Exercise</u>: Compute $\lim_{t \to \frac{7\pi}{16}} \vec{r}(t) = \langle (1+5\sin(20t))\cos(8t), (1+5\sin(20t))\sin(8t), \cos(20t) \rangle$